SHEP 93/94-15

Bottom-Tau Yukawa Unification in the Next-to-Minimal Supersymmetric Standard Model.

B. C. Allanach and S. F. King

Physics Department, University of Southampton
Southampton, SO9 5NH, U.K.

Abstract

We discuss the unification of the bottom quark and tau lepton Yukawa couplings within the framework of the next-to-minimal supersymmetric standard model. We compare the allowed regions of the m_t-$\tan\beta$ plane to those in the minimal supersymmetric standard model, and find that over much of the parameter space the deviation between the predictions of two models is small, and nearly always much less than the effect of current theoretical and experimental uncertainties in the bottom quark mass and the strong coupling constant. However over some regions of parameter space top-bottom Yukawa unification cannot be achieved. We also discuss the scaling of the light fermion masses and mixing angles, and show that to within current uncertainties the results of recent texture analyses performed for the minimal model also apply to the next-to-minimal model.

It was realized some time ago that the simplest grand unified theories (GUTs) based on SU(5) predict the Yukawa couplings of the bottom quark and the tau lepton to be equal at the GUT scale [1],

$$\lambda_b(M_{GUT}) = \lambda_\tau(M_{GUT}) \tag{1}$$

where $M_{GUT} \sim 10^{16}GeV$. [1] Assuming the effective low energy theory below M_{GUT} to be that of the minimal supersymmetric standard model (MSSM) the boundary condition in Eq.1 leads to a physical bottom to tau mass ratio m_b/m_τ in good agreement with experiment [2]. Spurred on by recent LEP data which is consistent with coupling constant unification, the relation in Eq.1 has recently been the subject of intense scrutiny using increasingly sophisticated levels of approximation [3], [4]. These analyses showed that for given values of the strong coupling constant $\alpha_3(M_Z)$ and m_b, there are only two allowed regions of $\tan\beta$ for each choice of top quark mass m_t: a small $\tan\beta$ branch and a large $\tan\beta$ branch. [2] However the results are strongly dependent on $\alpha_3(M_Z)$ and m_b, as well as GUT scale threshold effects.

The recent investigations of the relation in Eq.1 have focused on the MSSM. In this paper we shall instead assume that the effective low energy theory below the GUT scale is the next-to-minimal supersymmetric standard model (NMSSM) rather than the MSSM. The NMSSM [5] involves a single gauge singlet N and has the μ parameter set to zero, with its effect being replaced by terms in the superpotential like $\lambda N H_1 H_2$ and kN^3, where λ and k are dimensionless couplings particular to the NMSSM. The scalar component of N is assumed to develop a weak scale VEV, x, whose value will not directly enter our calculations, since N does not directly couple to quarks and leptons. The NMSSM is an alternative to the MSSM which is equally consistent with coupling constant unification, and is at least as well motivated as the minimal model. Since the renormalisation group (RG) equations of the heavy fermions involve the new couplings λ and k which may be quite large, there is no reason to expect the allowed regions of the $m_t - \tan\beta$ plane, consistent with Eq.1, to bear any resemblance to those in the MSSM. However, as it turns out the allowed regions in the NMSSM are quite similar to those in the MSSM although for large values of $k(M_{SUSY})$ and $\lambda(M_{SUSY})$ the large $\tan\beta$ branch cannot be achieved.

[1] The relation in Eq.1 is not exclusive to minimal $SU(5)$, but also applies to other simple GUTs such as $SO(10)$ and E_6, providing the Higgs doublets are embedded in the smallest representations, as in the minimal $SU(5)$ model.

[2] Recall that the MSSM is a two Higgs doublet model where H_1 is the Higgs doublet which gives mass to down-type quarks and charged leptons, while H_2 gives mass to up-type quarks. The superpotential contains a term $\mu H_1 H_2$, and the ratio of the two Higgs vacuum expectation values (VEVs) is $\tan\beta = v_2/v_1$.

Our procedure closely follows that of ref.[4]. We preface our discussion with a brief summary of our approach and approximations. For a grid of $\tan\beta$ and m_t values we determine the Yukawa couplings of the heavy fermions at the scale M_{SUSY}. The three gauge couplings g_i $(i = 1, 2, 3)$ at M_{SUSY} are determined by running them up from their measured values at M_Z. Having chosen values of λ and k at M_{SUSY} we run all the couplings up to $M_{GUT} = 10^{16}GeV$ using the SUSY RG equations and check if Eq.1 is satisfied to an accuracy of 0.1%, for all the values of $\tan\beta$ and m_t in the grid. Since our goal is to compare the results of the NMSSM to those of the MSSM at the same level of approximation, it is sufficient to work to one loop order in the RG equations between M_{SUSY} and M_{GUT}. Similarly, we take $M_{SUSY} = m_t$ for convenience, and ignore all low energy threshold effects i.e. assume all SUSY partners are degenerate with the top quark. However we shall consider the important effects of GUT scale threshold effects by considering

$$\lambda_b(M_{GUT}) = 0.9\lambda_\tau(M_{GUT}) \tag{2}$$

rather than Eq.1. In some models such as flipped SU(5) [6] Eq.2 may apply at tree-level.

The various couplings at $M_{SUSY} = m_t$ were determined as follows. The running masses of the fermions $m_f(\mu)$, where $f = b, \tau$ and μ is the scale, were determined by running them up from their mass shell values $m_f(m_f)$ with effective 3 loop QCD $\otimes$ 1 loop QED [7], [8]. This enables the Yukawa couplings to be determined at m_t by:

$$\lambda_t\left(m_t\right) = \frac{\sqrt{2}m_t\left(m_t\right)}{v\sin\beta} \tag{3}$$

$$\lambda_b\left(m_t\right) = \frac{\sqrt{2}m_b\left(m_b\right)}{\eta_b v\cos\beta} \tag{4}$$

$$\lambda_\tau\left(m_t\right) = \frac{\sqrt{2}m_\tau\left(m_\tau\right)}{\eta_\tau v\cos\beta}. \tag{5}$$

where

$$\eta_f = \frac{m_f\left(m_f\right)}{m_f\left(m_t\right)} \tag{6}$$

and the VEV $v = \sqrt{v_1^2 + v_2^2} = 246$ GeV. Clearly the Yukawa couplings at m_t depend on both $\alpha_3(M_Z)$ and $m_b(m_b)$. In the NMSSM the additional parameters $\lambda(m_t)$ and $k(m_t)$ are unconstrained, and may be regarded as additional free input parameters in our analysis. Finally the gauge couplings at $M_{SUSY} = m_t$ were determined from some input values at M_Z, by using the standard model RG equations (including 5 quark

flavours and no scalar fields). The input values were taken to be $\alpha_1(M_Z)^{-1} = 58.89$, $\alpha_2(M_Z)^{-1} = 29.75$, with $\alpha_3(M_Z) = 0.10 - 0.12$. Note that whereas the m_t referred to here is always the running one, it can be related as in [4] to the physical mass by

$$m_t^{phys} = m_t\left(m_t\right)\left[1 + \frac{4}{3\pi}\alpha_3\left(m_t\right) + O\left(\alpha_3^2\right)\right]. \tag{7}$$

Given the dimensionless couplings at $M_{SUSY} = m_t$, they are then run up to $M_{GUT} = 10^{16}GeV$ using the following SUSY RG equations relevant for the NMSSM, which we obtained in a straightforward way from ref.[9]. Including the full Yukawa matrices we find

$$\begin{aligned}
\frac{\partial U}{\partial t} &= \frac{U}{16\pi^2}\left[3\mathrm{Tr}\left(UU^\dagger\right) + 3U^\dagger U + D^\dagger D + \lambda^2 - \left(\frac{13}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)\right] \\
\frac{\partial D}{\partial t} &= \frac{D}{16\pi^2}\left[\mathrm{Tr}\left(3DD^\dagger + EE^\dagger\right) + U^\dagger U + 3D^\dagger D + \lambda^2 - \left(\frac{7}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)\right] \\
\frac{\partial E}{\partial t} &= \frac{E}{16\pi^2}\left[\mathrm{Tr}\left(3DD^\dagger + EE^\dagger\right) + 3E^\dagger E + \lambda^2 - \left(\frac{9}{5}g_1^2 + 3g_2^2\right)\right] \\
\frac{\partial \lambda}{\partial t} &= \frac{\lambda}{16\pi^2}\left[\mathrm{Tr}\left(3UU^\dagger + 3DD^\dagger + EE^\dagger\right) + 4\lambda^2 + 2k^2 - \left(\frac{3}{5}g_1^2 + 3g_2^2\right)\right] \\
\frac{\partial k}{\partial t} &= \frac{6k}{16\pi^2}\left[k^2 + \lambda^2\right],
\end{aligned} \tag{8}$$

where U, D and E are the up, down and charged lepton Yukawa matrices respectively and the GUT normalisation convention of g_1 has been used. Dropping small Yukawa couplings Eq.8 reduces to

$$\begin{aligned}
16\pi^2\frac{\partial \lambda_t}{\partial t} &= \lambda_t\left[6\lambda_t^2 + \lambda_b^2 + \lambda^2 - \left(\frac{13}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)\right] \\
16\pi^2\frac{\partial \lambda_b}{\partial t} &= \lambda_b\left[6\lambda_b^2 + \lambda_\tau^2 + \lambda_t^2 + \lambda^2 - \left(\frac{7}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)\right] \\
16\pi^2\frac{\partial \lambda_\tau}{\partial t} &= \lambda_\tau\left[\lambda_\tau^2 + 3\lambda_b^2 + \lambda^2 - \left(\frac{9}{5}g_1^2 + 3g_2^2\right)\right] \\
16\pi^2\frac{\partial \lambda}{\partial t} &= \lambda\left[4\lambda^2 + 2k^2 + 3\lambda_\tau^2 + 3\lambda_b^2 + 3\lambda_t^2 - \left(\frac{3}{5}g_1^2 + 3g_2^2\right)\right] \\
16\pi^2\frac{\partial k}{\partial t} &= 6k\left[\lambda^2 + k^2\right].
\end{aligned} \tag{9}$$

Our results are displayed in Fig.1 as contours in the $\tan\beta - m_t$ plane consistent with Eq.1. We take $\alpha_3(M_Z) = 0.11$, $m_b = 4.25GeV$ and the NMSSM parameters $\lambda(m_t)$ and $k(m_t)$ as indicated. The MSSM contour is shown for comparison and is indistinguishable from the NMSSM contour with $\lambda\left(m_t\right) = 0.1$ and $k\left(m_t\right) = 0.5$. In fact our plot for the MSSM based on 1-loop RG equations is very similar to the 2-loop result in ref.[4]. The deviation of the NMSSM contours from the MSSM

contour depends most sensitively on $\lambda(m_t)$ rather than $k(m_t)$. Two of the contours are shortened due to either λ or k blowing up at the GUT scale. For $\lambda(m_t) = 0.5$, $k(m_t) = 0.5$, no points in the $m_t - \tan\beta$ plane are consistent with Eq.1 Yukawa unification, while for $\lambda(m_t) = 0.1$, $k(m_t) = 0.1 - 0.5$ the contours are virtually indistinguishable from the MSSM contour. In general we find that for any of the current experimental limits on α_3 and m_b, the maximum value of $\lambda(m_t)$ or $k(m_t)$ is ~ 0.7 for a perturbative solution to Eq.1.

In Figs.2 and 3 we examine the effect of the experimental uncertainties in m_b and $\alpha_3(M_Z)$ as well as the theoretical uncertainties parameterised by Eq.2. In Fig.2, for a rather low value of $\alpha_3(M_Z) = 0.10$, the region between the two solid lines respects Eq.1 in the NMSSM with $\lambda(m_t) = 0.5$, $k = 0.1$, and $m_b = 4.1 - 4.4GeV$, while the dashed line satisfies Eq.2 for $m_b = 4.4GeV$. The corresponding dashed line which satisfies Eq.2 for $m_b = 4.1GeV$ occurs for $m_t < 100GeV$ and so is to the left of the figure. Clearly for this value of strong coupling, the error in the bottom quark mass turns the sharp contours in Fig.1 into thick bands, leading to no prediction of $\tan\beta$ for $m_t > 150GeV$. In Fig.3, for a higher value of $\alpha_3(M_Z) = 0.12$, the same uncertainties have a much less severe effect than in Fig.2, and larger values of the top quark mass are permitted. However the moral of Figs.2 and 3 is clear: the sharp contours of Fig.1 must not be taken too seriously given the present experimental and theoretical uncertainties.

So far we have discussed the heavy third family fermion masses only. Let us now extend our discussion of the NMSSM to include the light fermion masses and mixing angles. It is well known in the MSSM that to one loop order in the RG equations, the running of the physically relevant Yukawa eigenvalues and mixing angles can be expressed in simple terms as shown below,

$$\begin{aligned}
\left(\frac{\lambda_{u,c}}{\lambda_t}\right)_{M_{SUSY}} &= \left(\frac{\lambda_{u,c}}{\lambda_t}\right)_{M_{GUT}} e^{3I_t+I_b} \\
\left(\frac{\lambda_{d,s}}{\lambda_b}\right)_{M_{SUSY}} &= \left(\frac{\lambda_{d,s}}{\lambda_b}\right)_{M_{GUT}} e^{3I_b+I_t} \\
\left(\frac{\lambda_{e,\mu}}{\lambda_\tau}\right)_{M_{SUSY}} &= \left(\frac{\lambda_{e,\mu}}{\lambda_\tau}\right)_{M_{GUT}} e^{3I_\tau} \\
\frac{\mid V_{cb}\mid_{M_{GUT}}}{\mid V_{cb}\mid_{M_{SUSY}}} &= e^{I_b+I_t},
\end{aligned} \tag{10}$$

with identical scaling behaviour to V_{cb} of V_{ub}, V_{ts}, V_{td}, where

$$I_i = \int_{\ln M_{SUSY}}^{\ln M_{GUT}} \left(\frac{\lambda_i\,(t)}{4\pi} \right)^2 dt. \tag{11}$$

To a consistent level of approximation V_{us}, V_{ud}, V_{cs}, V_{cd}, V_{tb}, λ_u/λ_c, λ_d/λ_s and λ_e/λ_μ are RG invariant. The CP violating quantity J scales as V_{cb}^2. The Eqs. 10, 11 also apply to the NMSSM since the extra λ and k paramters cancel out of the RG equations in a similar way to the gauge contributions as can easily be seen from Eq.8. The only difference to these physically relevant quantities is therefore contained in I_τ, I_b and I_t.

In Figs.4-6 we plot the values of I_τ, I_b and I_t as evaluated along the contours in Figs.1-3 which satisfy Eq.1. Fig.4 illustrates the difference in the I_i quantities between the MSSM and the NMSSM for the contour in Fig.1 corresponding to $\lambda(m_t) = 0.5$, $k(m_t) = 0.1$. The NMSSM results are the upper lines of each pair, and it is clear that the deviation between the two models is small. In Fig.5 we plot the three I_i integrals for the NMSSM along the two solid contours of Fig.2, corresponding to a low value of $\alpha_3(M_Z) = 0.10$ and a range of $m_b = 4.1 - 4.4GeV$. The experimental uncertainty in m_b shown in Fig.5 clearly swamps the theoretical difference between the values of I_b and I_t for the NMSSM and the MSSM shown in Fig.4, for this choice of $\alpha_3(M_Z)$. In Fig.6 we plot the three I_i integrals along the two solid contours of Fig.3, corresponding to a higher value of $\alpha_3(M_Z) = 0.12$ and the same range of m_b. Again the large deviations in I_b and I_t due to the error in m_b swamp the theoretical differences between the two models. However, the values of I_τ in Figs.5,6 are quite robust, and the theoretical deviation in I_τ shown in Fig.4 is significant, and may play an important role in distinguishing between the two models.

We emphasise that the results of the I_i integrals shown in Figs.4-6 play a key role in determining the entire fermion mass spectrum via the scaling relations shown in Eq.10. The small deviation between the NMSSM and the MSSM results compared to the experimental uncertainties, means that the recent GUT scale texture analyses of the quark mass matrices which were performed for the MSSM are equally applicable to the NMSSM. For example, the recent Ramond, Roberts and Ross (RRR) [10] texture analysis is also based upon Eq.1 and assumes a Georgi-Jarlskog (GJ) [11], [12] ansatz for the charged lepton Yukawa matrices, although their results in the quark sector are insensitive to the lepton sector. It is clear that all the RRR results are immediately applicable to the NMSSM without further ado since the only difference between the two models enters through the scaling integrals I_i whose deviation we have shown to

be negligible compared to the experimental errors.

Finally in Fig.7 we examine the question of full top-bottom-tau Yukawa unification in the NMSSM. Third family Yukawa unification has recently been studied in some detail in the MSSM [13] and occurs theoretically in minimal $SO(10)$ [14] and $SU(4) \otimes SU(2)^2$ [15] models. Fig.7 shows the values of the Yukawa couplings evaluated at the GUT scale for values of $\tan\beta$ along two of the contours in Fig.1 corresponding to the NMSSM with $\lambda(m_t) = 0.5$ and $k(m_t) = 0.1, 0.4$. The tau Yukawa coupling is of course equal to the bottom Yukawa coupling at the GUT scale and so is not labelled explicitly in this figure. In Fig.1 it was observed that the high k lines are shorter than the others and consequently the large $\tan\beta$ regions cannot be achieved. The reason is clear from Fig.7, which shows that $k(M_{GUT})$ blows up for large $\tan\beta$. This is because, as can be seen from Eq.9, k scales quickly and readily becomes nonperturbative for $k(m_t) > 0.5$. The stunted lines in Fig.7 mean that, for $k(m_t) = 0.4$ and the other parameters as assumed in the figure, top-bottom Yukawa unification cannot be achieved. The longer lines in Fig.7 show that, for $k(m_t) = 0.1$ and all the other parameters unchanged, larger values of $\tan\beta$ may be achieved and top-bottom unification, which occurs for $\tan\beta \approx 50$ for this set of parameters, is once again possible.

In conclusion, we have discussed the unification of the bottom quark and tau lepton Yukawa couplings within the framework of the NMSSM. By comparing the allowed regions of the m_t-$\tan\beta$ plane to those in the MSSM we find that over much of the parameter space the deviation between the predictions of two models which is controlled by the parameter λ is small, and always much less than the effect of current theoretical and experimental uncertainties in the bottom quark mass and the strong coupling constant. We have also discussed the scaling of the light fermion masses and mixing angles, and shown that to within current uncertainties, the results of recent quark texture analyses [10] performed for the minimal model also apply to the next-to-minimal model. There are however two distinguishing features of the NMSSM. Firstly, the scaling of the charged lepton masses will be somewhat different, depending on λ and k. Although this will not affect the quark texture analysis of RRR, it may affect the success of the GJ ansatz [11], [12] for example. Secondly, the larger $\tan\beta$ regions may not be accessible in the NMSSM for large values of λ and k, so that full Yukawa unification may not be possible in this case.

References

[1] M. S. Chanowitz, J. Ellis and M. K. Gaillard, Nucl. Phys. **B128** (1977) 506; A. J. Buras, J. Ellis, M. K. Gaillard and D. V. Nanopoulos, Nucl. Phys. **B135** (1978) 66.

[2] M. B. Einhorn and D. R. T. Jones, Nucl. Phys. **B196** (1982) 475; J. Ellis, D. V. Nanopoulos and S. Rudaz, Nucl. Phys. **B202** (1982) 43.

[3] H. Arason *et al*, Phys. Rev. Lett. **67** (1991) 2933; *ibid* Phys. Rev. **D47** (1993) 232; P. Langacker and N. Polonsky, UPR-0556T.

[4] V.Barger, M.S.Berger and P.Ohmann, Phys. Rev. **D47** (1993) 1093.

[5] P. Fayet, Nucl. Phys. **B90** (1975) 104; J. Ellis, J. Gunion, H. Haber, L. Roszkowski and F. Zwirner, Phys. Rev. **D39** (1989) 844; T. Elliott, S. King and P. White, Radiative Corrections to Higgs Boson Masses in the Next-to-Minimal Supersymmetric Standard Model, SHEP 92/93-21, hep-ph/9308309, Phys. Rev. **D** (to appear).

[6] I. Antoniadis, J. Ellis, J. Hagelin and D. Nanopoulos, Phys. Lett. **B194** (1987) 231.

[7] S.G.Gorishny, A.L.Kataev, S.A.Larin and L.R.Surgaladze, Mod. Phys. Lett. **A32**, Vol. 5, (1990) 2703; O.V.Tarasov, A.A.Vladimirov and A.Yu.Zharkov, Phys. Lett. **B93** (1980) 429; S.G.Korishny, A.L.Kataev and S.A.Larin, Phys. Lett. **B135** (1984) 457.

[8] L.Hall, Nucl. Phys. **B178** (1981) 75.

[9] S.P.Martin and M.T.Vaughn, NUB-3081-93TH, hep-ph 9311340.

[10] P. Ramond, R. Roberts and G. Ross, RAL-93-010, UFIFT-93-06.

[11] H. Georgi and C. Jarlskog, Phys. Lett. **B86** (1979) 297;

[12] S. Dimopoulos, L. Hall and S. Raby, Phys. Rev. **D45** (1992) 4192.

[13] B. Ananthanarayan, G. Lazarides and Q. Shafi, Phys. Rev. **D44** (1991) 1613; H. Arason *et al*, Phys. Rev. Lett. **67** (1991) 2933; L. Hall, R. Rattazzi and U. Sarid, LBL preprint 33997.

[14] G. Anderson, S. Raby, S. Dimopoulos, L. Hall and G. Starkman, LBL preprint 33531.

[15] I. Antoniadis and G. K. Leontaris, Phys. Lett. **B245** (1989) 333; S. King, Southampton preprint SHEP 93/94-08 (Phys. Lett. B, to appear).

Figure Captions

Figure 1. Contours in the $m_t - \tan\beta$ plane over which the bottom-tau unification condition Eq.1 is satisfied in the MSSM and the NMSSM. Central values of $\alpha_3(M_Z) = 0.11$ and $m_b = 4.25$ GeV are assumed, and NMSSM contours for various $\lambda(m_t)$ and $k(m_t)$ values are shown.

Figure 2. Contours in the $m_t - \tan\beta$ plane over which Eq.1 is satisfied in the NMSSM with $\lambda(m_t) = 0.5$ and $k(m_t) = 0.1$, for a low value of $\alpha_3\,(M_Z) = 0.10$. The region between the two solid lines is for exact bottom-tau unification, with $m_b = 4.1 - 4.4 GeV$. The dashed line satisfies $\lambda_b = 0.9\lambda_\tau$ as in Eq.2 for $m_b = 4.4 GeV$.

Figure 3. Contours in the $m_t - \tan\beta$ plane over which Eq.1 is satisfied in the NMSSM with $\lambda(m_t) = 0.5$ and $k(m_t) = 0.1$, for a high value of $\alpha_3\,(M_Z) = 0.12$. The region between the two solid lines is for exact bottom-tau unification, with $m_b = 4.1 - 4.4 GeV$. The region between the two dashed lines satisfies $\lambda_b = 0.9\lambda_\tau$ as in Eq.2 for $m_b = 4.1 - 4.4 GeV$.

Figure 4. The I_f integrals defined in the text as evaluated along two of the bottom-tau unification contours in Fig.1. As in Fig.1, $\alpha_3(M_Z) = 0.11$ and $m_b = 4.25$ GeV. The pairs of lines shown in this figure correspond to the MSSM (lower lines) and the NMSSM with $\lambda(m_t) = 0.5$, $k(m_t) = 0.1$ (upper lines).

Figure 5. The I_f integrals defined in the text as evaluated along the two exact bottom-tau unification contours of Fig.2. As in Fig.2, $\alpha_3(M_Z) = 0.10$, $\lambda_b(m_t) = 0.5$, $k(m_t) = 0.1$. The shorter lines in this figure correspond to $m_b = 4.1 GeV$, the longer lines to $m_b = 4.4 GeV$.

Figure 6. The I_f integrals defined in the text as evaluated along the two exact bottom-tau unification contours of Fig.3. As in Fig.3, $\alpha_3(M_Z) = 0.12$, $\lambda_b(m_t) = 0.5$, $k(m_t) = 0.1$. The shorter lines in this figure correspond to $m_b = 4.4 GeV$, the longer lines to $m_b = 4.1 GeV$.

Figure 7. The various NMSSM couplings evaluated at the GUT scale, corre-

sponding to the two bottom-tau unification contours in Fig.1 with $\lambda(m_t) = 0.5$. The longer lines in this figure are from the $k(m_t) = 0.1$ line, and exhibit top-bottom unification for $\tan\beta \approx 50$. The shorter ones are from the $k(m_t) = 0.4$ curve, and show that top-bottom unification cannot be achieved since k blows up before a sufficiently large $\tan\beta$ can be achieved.